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THE AVERAGE SPEED OF A FAST VEHICLE MOVING IN A STREAM OF SLOW VEHICLES

by

10 Zeev Barzily

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This paper studies the average speed of a fast test car moving in a stream of slow vehicles on a two-lane highway. Arrivals of the slow vehicles are assumed to follow a Poisson process and the test car arrives independently of the slow vehicles. The highway is assumed to consist of sections in which passing is possible and sections in which passing is impossible; the lengths of these sections are random variables. Two passing mechanisms are studied: the first assumes that the duration of a passing maneuver is a random variable while in the second passings are instantaneous.

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THE AVERAGE SPEED OF A FAST VEHICLE MOVING IN A STREAM OF SLOW VEHICLES

by

Zeev Barzily

1. Introduction

This paper studies the speed of a fast test car moving in a stream of slow vehicles in a two-lane two-way highway. Let "our direction" designate the direction in which the test-car is traveling. We assume that the highway consists of two alternating sections, sections of Type I in which passing is possible and sections of Type II in which passing is impossible. The highway is assumed to begin with a Type I section. Let X_i and Y_i denote the length of the ith Type I and Type II sections, respectively. We assume that $X_1, X_2, \ldots$ are i.i.d. distributed according to a continuous c.d.f. G with an expectation $E[X]$. The random variables $Y_1, Y_2, \ldots$ are also assumed to be i.i.d.; they are distributed according to a continuous c.d.f. H with an expectation $E[Y]$. The functions G and H depend on road conditions and on traffic moving in the opposite direction. We assume that the slow vehicles moving in our direction arrive at the highway according to a Poisson process with parameter λ_1. These zero size vehicles always maintain their free speed v_1. The test car arrives at the highway independently of the slow vehicles, and it has a free speed v_2 $(v_2 > v_1)$.

As the test car travels along the road it occasionally comes up against slow vehicles. At these points the test car's driver may decide to pass the slow vehicle or otherwise to reduce his speed immediately to v_1 and to follow the slow vehicle for a while. The decisions to pass are made either at the points where the test car comes up against slow vehicles in Type I sections, or at the beginnings of the Type I sections to which the test car arrives traveling behind a slow vehicle. We assume that the driver's decision is dependent on the distance from the decision point to the end of the Type I section and independent of the distance he has already been following the slow vehicle. Two passing mechanisms are studied. The first mechanism assumes that at each decision point the test car's driver samples a required passing distance W_1 from a c.d.f. B . If the distance to the end of the Type I section exceeds W_1 then the passing will take place W_1 units of distance from the decision point; otherwise the test car continues following the slow vehicle at least until the beginning of the next Type I section. In the second passing mechanism, the driver samples a r.v. W_2 from a c.d.f. C . He passes instantaneously at the decision point if the distance to the end of the Type I section exceeds W_2 ; otherwise he follows the slow vehicle at least until the beginning of the next Type I section.

In the present paper we use some of the results obtained by Barzily and Rubinovitch [1] in 1977. Passing mechanism number one requires the analysis of a model close to the one analyzed by Rubinovitch and Grinstein [3] in 1973 and by Sivazlian [4] in 1971. Some of the results in the paper are extensions of results derived by Galin and Epstein [2] in 1974. For more detailed information on models for traffic in two-lane roads, see [2] and [1].

The paper is comprised of five sections. In Sections 2 and 3 we discuss the traveling of the test car in Type I sections under the first

and the second passing mechanisms, respectively. In Section 4 we determine the test car's average speed. Section 5 is a short summary.

2. The Movement of the Test Car in a Type I Section Under Passing Mechanism Number One

In this section we discuss the movement of the test car in the ith Type I section under passing mechanism number one. Let us assume temporarily, for the convenience of the analysis, that this section is infinitely long. Define that the test car is in state i (i=1,2) at a point along the road if it is moving there at speed v_i . Denote by Z_{nj} the distance the test car travels at a speed v_n for the jth time since entering the ith section. It was shown earlier in [1] that while moving at a speed v_2 , the distance from the test car to the preceding slow vehicle is an exponential random variable with a parameter λ_1/v_1 . From this result we obtain that $Z_{2,1}, Z_{2,2}, \ldots$ are i.i.d. random variables distributed according to an exponential distribution function with parameter $\alpha = \lambda_1(1/v_1 - 1/v_2)$. The random variables $Z_{1,1}, Z_{1,2}, Z_{1,3}$ are (by assumption) i.i.d. random variables distributed according to a c.d.f. B . Let us now denote by $T(x)$ the time it takes the test car to arrive at a distance x from the beginning of the section; let $M_1(x)$ denote the state of the car at that point, and define $U(x) = x/v_1 - T(x)$. A realization of $T(x)$ and the corresponding $U(x)$ for $M_1(0) = 2$ is given in Figure 1. It is easy to analyze $T(x)$ using the analysis of $U(x)$.

Denote

$$q_{2j}(x,u) = P[U(x)\leq u, M_1(x)=j \mid M_1(0)=2] , \quad j=1,2 ,$$

$$Q^*_{2j}(\theta,\xi) = \int_{x=0}^{\infty} \int_{u=0}^{\infty} e^{-\theta x} e^{-\xi u} d_u q_{2j}(x,u)dx ,$$

and

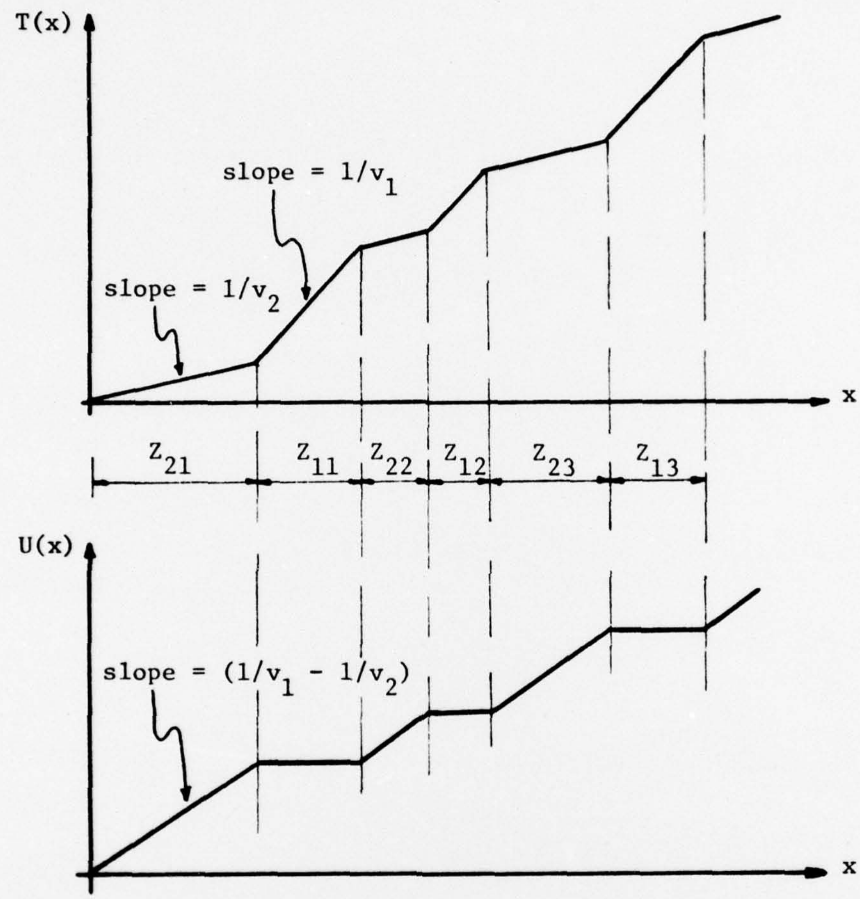

Figure 1. A typical realization of $T(x)$ and $U(x)$ for $M_1(0) = 2$.

$$q_{2j}(x,\cdot) \;=\; P[M_1(x)=j \mid M_1(0)=2] \; .$$

Denoting $\beta = 1/v_1 - 1/v_2$, we obtain

$$q_{21}(x,u) = \begin{cases} \int_0^{u/\beta} \alpha e^{-\alpha z} \, [1 - B(x-z)]dz \; + \\[2ex] \int_{z=0}^{u/\beta} \int_{y=0}^{x-z} \alpha e^{-\alpha z} \, q_{21}(x-z-y,\; u-z\beta)dB(y)dz \;, & 0 < u < x\beta \; ; \\[2ex] q_{21}(x,\cdot) \;, & x\beta \leq u \end{cases}$$

hence,

$$Q^*_{21}(\theta,\xi) \;=\; \frac{[1 - B^*(\theta)]\alpha}{\theta[\alpha + \theta + \xi\beta - \alpha B^*(\theta)]} \; , \tag{2.1}$$

where

$$B^*(\theta) \;=\; \int_{x=0}^{\infty} e^{-\theta x} \, dB(x) \; .$$

To obtain $Q^*_{22}(\theta,\xi)$, we notice that

$$q_{22}(x,u) = \begin{cases} \int_{z=0}^{u/\beta} \int_{y=0}^{x-z} \alpha e^{-\alpha z} \, q_{22}(x-z-y,\; u-z\beta)dB(y)dz \;, & 0 < u < x\beta \\[2ex] \int_{z=0}^{u/\beta} \int_{y=0}^{x-z} \alpha e^{-\alpha z} \, q_{22}(x-z-y,\; u-z\beta)dB(y)dz + e^{-\alpha z} \;, & u = x\beta \\[2ex] q_{22}(x,\cdot) \;, & u > x\beta \end{cases} \; ;$$

hence,

$$Q^*_{22}(\theta,\xi) \;=\; [\alpha + \theta + \xi\beta - \alpha B^*(\theta)]^{-1} \; . \tag{2.2}$$

Now we denote

$$p_{ij}(x,t) \;=\; P[T(x)\leq t,\; M_1(x)=j \mid M_1(0)=i] \;, \quad i,j = 1,2$$

and

$$P^*_{ij}(\theta,\xi) = \int_{x=0}^{\infty} \int_{t=0}^{\infty} e^{-\theta x} e^{-\xi t} d_t p_{ij}(x,t)dx , \quad i,j = 1,2; \ \theta,\xi > 0 .$$

To determine $P^*_{ij}(\theta,\xi)$ we realize that

$$p_{2j}(x,t) =$$

$$P\left[U(x) > \frac{x}{v_1} - t,\ M_1(x)=j \mid M_1(0)=2\right] + P\left[U(x) = \frac{x}{v_1} - t,\ M_1(x)=j \mid M_1(0)=2\right] ;$$

hence we obtain from (2.1)

$$P^*_{21}(\theta,\xi) = \frac{\alpha\left[1 - B^*\left(\theta + \frac{\xi}{v_1}\right)\right]}{\left(\theta + \frac{\xi}{v_1}\right)\left[\alpha + \theta + \frac{\xi}{v_2} - \alpha B^*\left(\theta + \frac{\xi}{v_1}\right)\right]} , \tag{2.3}$$

and from (2.2) we get

$$P^*_{22}(\theta,\xi) = \left[\alpha + \theta + \frac{\xi}{v_2} - \alpha B^*\left(\theta + \frac{\xi}{v_1}\right)\right]^{-1} . \tag{2.4}$$

Now we determine $P^*_{1j}(\theta,\xi)$, $j=1,2$. For $j=1$ we have

$$p_{11}(x,t) = \begin{cases} 0 & , \ t \leqq \ell/v_2 \\ \int_{y=0}^{(t-x/v_2)/\beta} p_{21}(x-y,\ t-y/v_1)dB(y) & , \ x/v_2 < t < x/v_1 \\ \int_{y=0}^{(t-x/v_2)/\beta} p_{21}(x-y,\ t-y/v_1)dB(y) + 1-B(x) & , \ t = x/v_1 \\ p_{11}(x,\cdot) & , \ t > x/v_1 \end{cases} ;$$

hence,

$$P^*_{11}(\theta,\xi) = \frac{1 - B^*\left(\theta + \frac{\xi}{v_1}\right)}{\theta + \frac{\xi}{v_1}} + \frac{\alpha\left[1 - B^*\left(\theta + \frac{\xi}{v_1}\right)\right] B^*\left(\theta + \frac{\xi}{v_1}\right)}{\left(\theta + \frac{\xi}{v_1}\right)\left[\alpha + \theta + \frac{\xi}{v_2} - \alpha B^*\left(\theta + \frac{\xi}{v_1}\right)\right]} , \tag{2.5}$$

and for $j=2$ we obtain

$$p_{12}(x,t) = \begin{cases} 0 & , \quad t \leq x/v_2 \\ \int_{y=0}^{(t-x/v_2)/\beta} p_{22}(x-y,\ t-y/v_1)\,dB(y) & , \quad x/v_2 < t \leq x/v_1 \ ; \\ p_{12}(x,\cdot) & , \quad x/v_1 < t \end{cases}$$

hence,

$$P^*_{12}(\theta,\xi) = \frac{B^*\left(\theta + \frac{\xi}{v_1}\right)}{\alpha + \theta + \frac{\xi}{v_2} - \alpha B^*\left(\theta + \frac{\xi}{v_1}\right)} . \qquad (2.6)$$

Let us now denote by $T_{ij}(x)$ the time spent by the test car in a Type I section of length x given that $M_1(0)=i$ and $M_1(x)=j$. The c.d.f. of $T_{ij}(x)$ satisfies

$$P[T_{ij}(x)\leq t] = \frac{P[T(x)\leq t,\ M_1(x)=j \mid M_1(0)=i]}{P[M_1(x)=j \mid M_1(0)=i]} ;$$

hence, denoting by $R^*_{ij}(x,\xi)$ the inverse Laplace transform of $P^*_{ij}(\theta,\xi)$ and letting $r_{ij}(x) = P[M_1(x)=j \mid M_1(0)=i]$, we obtain

$$E[T_{ij}(x)] = \left(\frac{\partial}{\partial\xi} R^*_{ij}(x,0)\right) / r_{ij}(x) . \qquad (2.7)$$

The probabilities $r_{ij}(x)$, $i,j = 1,2$, satisfy $r_{ij}(x) = R^*_{ij}(x,0)$. To invert $P^*_{ij}(\theta,\xi)$ we have to specify the c.d.f. B . We assume that B *is the exponential distribution function with parameter* η . The inversion can be carried out for other distribution functions, but the expressions are likely to be very messy. Using tables of Laplace transforms we obtain

$$r_{21}(x) = \alpha\left[1 - \exp\left(-(\alpha+\eta)x\right)\right]/(\alpha+\eta) , \qquad (2.8)$$

$$r_{11}(x) = \left[\alpha + \eta \exp\left(-(\alpha+\eta)x\right)\right]/(\alpha+\eta) , \qquad (2.9)$$

$$r_{21}(x)E[T_{21}(x)] = \alpha\Big[x(\alpha/v_1 + \eta/v_2) + (\eta-\alpha)\beta\big(1 - \exp\big(-(\alpha+\eta)x\big)\big)/(\alpha+\eta)$$
$$- x(\eta/v_1 + \alpha/v_2)\exp\big(-(\alpha+\eta)x\big)\Big]/(\alpha+\eta)^2 , \quad (2.10)$$

$$r_{22}(x)E[T_{22}(x)] = \Big[x(\alpha\eta/v_1 + \eta^2/v_2) + 2\big(1 - \exp\big(-(\alpha+\eta)x\big)\big)\alpha\eta\beta/(\eta+\alpha)$$
$$+ x\exp\big(-(\alpha+\eta)x\big)(\alpha\eta/v_1 + \alpha^2/v_2)\Big]/(\alpha+\eta)^2 , \quad (2.11)$$

$$r_{11}(x)E[T_{11}(x)] = \Big[x(\alpha^2/v_1 + \alpha\eta/v_2) + 2\big(1 - \exp\big(-(\alpha+\eta)x\big)\big)\alpha\eta\beta/(\eta+\alpha)$$
$$+ x\exp\big(-(\alpha+\eta)x\big)(\eta^2/v_1 + \eta\alpha/v_2)\Big]/(\alpha+\eta)^2 , \quad (2.12)$$

$$r_{12}(x)E[T_{12}(x)] = r_{21}(x)E[T_{21}(x)]\eta/\alpha . \quad (2.13)$$

3. The Movement of the Test Car in a Type I Section under Passing Mechanism Number Two

In this section we derive results similar to those of Section 2 when the passing is instantaneous upon the test car driver's decision to pass. We start with the analysis of the case where $M_1(0)=2$ and assume that $X_i = x$. The distribution of $T_{22}(x)$ can easily be determined because $M_1(x)=2$ means here that the test car is unimpeded in the ith Type I section; hence,

$$T_{22}(x) = x/v_2 , \text{ with probability one.} \quad (3.1)$$

Now we assume without loss of generality that the test car arrives at the entrance of the ith Type I section at time zero. To obtain $\{T_{21}(x)\leq t\}$, $x/v_2 < t \leq x/v_1$, the test car has to be unimpeded by slow vehicles arriving at this section during $\big(-(x/v_1 - t), 0\big]$. Let $J(x,t)$ denote the number of slow vehicles arriving at the ith Type I section during $\big(-(x/v_1 - t), 0\big]$. It is known that given that $J(x,t) = n$, then the epochs of the arrivals are independent and uniformly distributed on $\big(-(x/v_1 - t), 0\big]$. Consequently we obtain

$$P[T(x)\leq t \mid M_1(0)=2,\ J(x,t)=n] = \left[\int_0^{(x/v_1)-t} C\left((t+y-x/v_2)/\beta\right) \frac{dy}{(x/v_1)-t}\right]^n ,$$

where $C\left((x+y-x/v_2)/\beta\right)$ is the probability that the test car passes immediately a slow vehicle that arrives at the section in $\left(-(x/v_1 - t) + y\right)$. Since $J(x,t)$ is a Poisson random variable with parameter $\lambda_1(x/v_1 - t)$, we obtain

$$P[T(x)\leq t \mid M_1(0)=2] = \exp\left[-\lambda_1\left(x/v_1 - t - \int_0^{(x/v_1)-t} C\left((t+y-x/v_2)/\beta\right) dy\right)\right] . \tag{3.2}$$

From (3.2) we obtain

$$r_{21}(x) = P[T(x)>x/v_2 \mid M_1(0)=2] = 1 - \exp\left[-\lambda_1\left(x\beta - \int_0^{x\beta} C(y/\beta)dy\right)\right] , \tag{3.3}$$

$$r_{22}(x)E[T_{22}(x)] = \exp\left[-\lambda_1\left(x\beta - \int_0^{x\beta} C(y/\beta)dy\right)\right]x/v_2 , \tag{3.4}$$

and

$$r_{21}(x)E[T_{21}(x)] = \int_{x/v_2}^{x/v_1} t\, d_t\left(P[T(x)\leq t \mid M(0)=2]\right) . \tag{3.5}$$

Now we turn to determine the results associated with $M_1(0)=1$. The derivation is based on the fact that

$$P[T(x)\leq t \mid M_1(0)=1] = \begin{cases} C(x)P[T(x)\leq t \mid M_1(0)=2] , & x/v_2 \leq t < x/v_1 \\ 1 , & t = x/v_1 \end{cases} ,$$

from which we obtain

$$r_{11}(x) = C(x)P[T(x)>x/v_2 \mid M_1(0)=2] + \left(1 - C(x)\right) , \tag{3.6}$$

$$r_{11}(x)E[T_{11}(x)] = C(x)r_{21}(x)E[T_{21}(x)] + \left(1 - C(x)\right)x/v_1 , \tag{3.7}$$

and

$$r_{12}(x)E[T_{12}(x)] = C(x)r_{22}(x)E[T_{22}(x)] . \tag{3.8}$$

4. The Test Car's Average Speed

In this section we use the results of Sections 2 and 3 to obtain the test car's average speed under the two passing mechanisms. As we follow the test car's journey along the road we realize that its state at the beginnings of the Type I sections forms a Markov chain. The analysis is based on this property.

To complement the results on the movement of the test car in a Type I section we need to have similar results on the travel in Type II sections. For this purpose we denote by $S_k(y)$, $k=1,2$, the time it takes the test car to travel y units of length along a Type II section given that $M_2(0)=k$ [$M_2(u)$ designates the state of the test car u units of length from the beginning of the Type II section]. We also denote

$$a_{ij}(y) = P[M_2(y)=j \mid M_2(0)=i] .$$

Here again we use the property that while the test car is in state 2 the distance between it and its preceding slow vehicle is an exponential random variable (parameter λ_1/v_1) and obtain

$$S_1(y) = y/v_1 \text{ with probability one,} \tag{4.1}$$

$$a_{11}(y) = 1 , \tag{4.2}$$

$$a_{22}(y) = \exp(-\lambda_1 y\beta) , \tag{4.3}$$

$$P[S_2(y)\leq s] = \exp[-\lambda_1(y/v_1 - s)] , \quad y/v_2 \leq s \leq y/v_1 . \tag{4.4}$$

For a more detailed derivation of (4.3) and (4.4), see [1]. From (4.4) we obtain

$$E[S_2(y)] = y/v_1 - [1 - \exp(-\lambda_1 y\beta)]/\lambda_1 . \tag{4.5}$$

We are now in a position where we can sum up the results derived so far to obtain the test car's average speed. To this end, let N_{ik} be the number of Type I sections to which the test car arrives at a speed v_k, $k=1,2$, while traveling up to the end of the ith Type II section. Denote by $\tau_k(j)$, $j=1,\ldots,N_{ik}$, the time it takes the test car to travel along a road section that consists of a Type I section and its following Type II section, given that the Type I section is the jth to which it arrives at speed v_k. The average speed at the end of the jth Type II section is given by

$$\bar{V}_j = \frac{\sum_{i=1}^{2} \sum_{k=1}^{N_{ji}} \tau_i(k)}{\sum_{i=1}^{j} (X_i+Y_i)} . \tag{4.6}$$

We will determine

$$\lim_{j\to\infty} \bar{V}_j .$$

From (4.6) we obtain

$$\lim_{j\to\infty} \bar{V}_j = \lim_{j\to\infty} \frac{1}{\frac{1}{j}\sum_{k=1}^{j} (X_k+Y_k)} \sum_{i=1}^{2} \lim_{j\to\infty} \frac{\sum_{k=1}^{N_{ji}} \tau_i(k)}{N_{ji}} \lim_{j\to\infty} \frac{N_{ji}}{j} . \tag{4.7}$$

The RHS of (4.7) calls for the use of the strong law of large numbers. Applying this law yields

$$\lim_{j\to\infty} \frac{1}{\frac{1}{j}\sum_{k=1}^{j} (X_k+Y_k)} = \frac{1}{E[X] + E[Y]} , \tag{4.8}$$

$$\lim_{j\to\infty} \frac{\sum_{k=1}^{N_{ji}} \tau_i(k)}{N_{ji}} = \sum_{\ell=1}^{2} \left\{ E_X\{r_{i\ell}(X) E[T_{i\ell}(X)]\} + E[r_{i\ell}(X)]\, E_Y\{E[S_\ell(Y)]\} \right\} , \tag{4.9}$$

and

$$\lim_{j\to\infty} \frac{N_{ji}}{j} = \pi_i , \tag{4.10}$$

where $\pi = (\pi_1, \pi_2)$ is the invariant distribution for the Markov chain of the state of the test car upon arrivals at Type I sections. The vector π is obtained as follows. Define

$$\gamma_{ij} = E[r_{ij}(X)] \; ; \quad \Gamma = \{\gamma_{ij}\}$$

and

$$\phi_{ij} = E[a_{ij}(Y)] \; ; \quad \Phi = \{\phi_{ij}\} ,$$

then π satisfies

$$\pi(\Gamma\Phi) = \pi$$

and

$$\sum_{i=1}^{2} \pi_i = 1 .$$

5. Summary

In the present paper we determined the average speed of a fast test car that is moving in a stream of slow vehicles. Two passing mechanisms were studied. The first mechanism assumes that after a driver decides to pass he still spends some time before reaching his free speed. The second mechanism, on the other hand, assumes instantaneous passing upon making the decision to pass. The first mechanism seems more realistic when the X's are small with respect to the Y's . Here the passings occur mainly at the beginnings of Type I sections and the test car has to accelerate before passing. The second mechanism seems more realistic

when the X's are large with respect to the Y's and the test car comes up against slower vehicles mainly in Type I sections. The realism of the two mechanisms may also depend on road conditions and traffic congestion. We could have easily added a third passing mechanism that is a combination of the first two--instantaneous passing inside Type I sections and passing according to mechanism number one upon arrival at the beginning of a Type I section. However, we do not think that this addition makes a substantial contribution on top of the other two.

Finally, we would like to note that the current model cannot be used in cases where traffic is heavy because in these cases the assumption that slow vehicles arrive according to a Poisson process is not suitable.

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- San Francisco

Naval Research Lab
- Tech Info Div
- Library, Code 2029 (ONRL)

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- Philadelphia, Pa.
- Hyattsville, Md.

Naval Ship Res & Dev Center

Naval Sea Systems Command
- Tech Library
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Naval Supply Systems Command
- Library
- Capt W. T. Nash

Naval War College Library
- Newport

BUPERS Tech Library

FMSO

Integrated Sea Lift Study

USN Ammo Depot Earle

USN Postgrad School Monterey
- Library
- Dr. Jack R. Borsting
- Prof C. R. Jones

US Marine Corps
- Commandant
- Deputy Chief of Staff, R&D

Marine Corps School Quantico
- Landing Force Dev Ctr
- Logistics Officer

Armed Forces Industrial College

Armed Forces Staff College

Army War College Library
- Carlisle Barracks

Army Cmd & Gen Staff College

US Army HQ
- LTC George L. Slyman
- Army Trans Mat Command

Army Logistics Mgmt Center
- Fort Lee

Commanding Officer, USALDSRA
- New Cumberland Army Depot

US Army Inventory Res Ofc
- Philadelphia

HQ, US Air Force
- AFADS-3

Griffiss Air Force Base
- Reliability Analysis Center

Maxwell Air Force Base Library

Wright-Patterson Air Force Base
- HQ, AF Log Command
- Research Sch Log

Defense Documentation Center

National Academy of Science
- Maritime Transportation Res Board Library

National Bureau of Standards
- Dr E. W. Cannon
- Dr Joan Rosenblatt

National Science Foundation

National Security Agency

WSEG

British Navy Staff

Logistics, OR Analysis Establishment
- National Defense Hdqtrs, Ottawa

American Power Jet Co
- George Chernowitz

ARCON Corp

General Dynamics, Pomona

General Research Corp
- Dr Hugh Cole
- Library

Planning Research Corp
- Los Angeles

Rand Corporation
- Library

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- Prof B. V. Dean
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- Prof R. E. Bechhofer
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Cowles Foundation for Research
- Library
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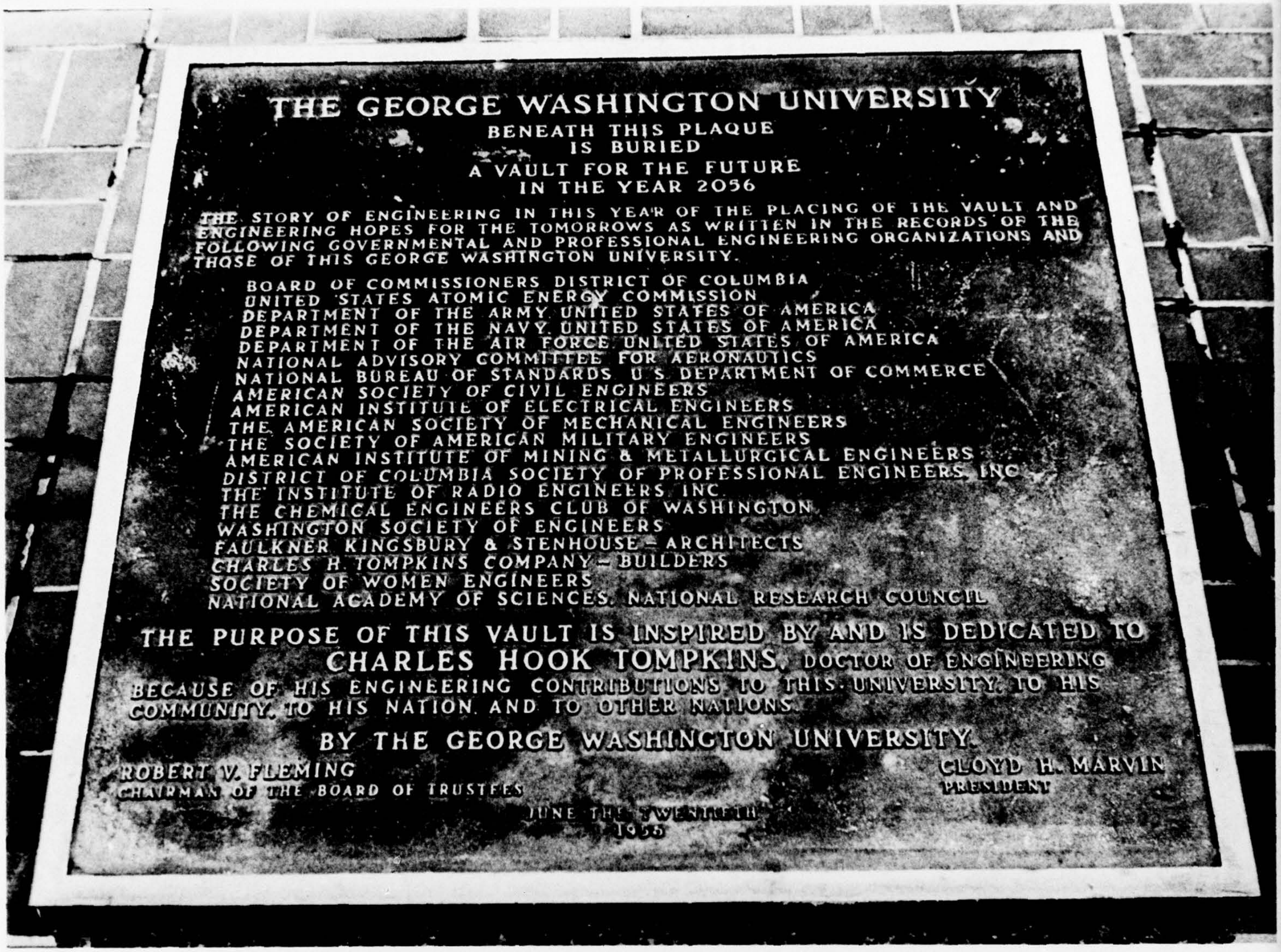

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